

Einstein's Photon Concept Quantified by the Bohr Model of the Photon

Geoffrey Hunter*, Marian Kowalski† and Camil Alexandrescu‡

York University, Toronto, Ontario, Canada M3J 1P3

February 1, 2008

Abstract

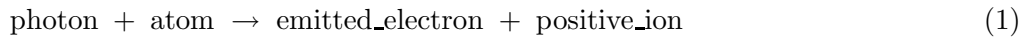
The photon is modeled as a monochromatic solution of Maxwell's equations confined as a soliton wave by the principle of causality of special relativity. The soliton travels rectilinearly at the speed of light. The solution can represent any of the known polarization (spin) states of the photon. For circularly polarized states the soliton's envelope is a circular ellipsoid whose length is the observed wavelength (λ), and whose diameter is λ/π ; this envelope contains the electromagnetic energy of the wave ($h\nu = hc/\lambda$). The predicted size and shape is confirmed by experimental measurements: of the sub-picosecond time delay of the photo-electric effect, of the attenuation of undiffracted transmission through slits narrower than the soliton's diameter of λ/π , and by the threshold intensity required for the onset of multiphoton absorption in focussed laser beams. Inside the envelope the wave's amplitude increases linearly with the radial distance from the axis of propagation, being zero on the axis. Outside the envelope the wave is evanescent with an amplitude that decreases inversely with the radial distance from the axis. The evanescent wave is responsible for the observed double-slit interference phenomenon.

1 Einstein's Concept of the Photon

In 1905 Einstein published a celebrated paper popularly known as “the photo-electric paper”, for which he was awarded the Nobel prize some 16 years later [1]. While this paper explains other physical phenomena in addition to the photo-electric effect, the unifying concept is that:

“The energy in a beam of light is not uniformly distributed as in a classical plane wave, but is localized in packets of electromagnetic radiation, each packet having an energy $h\nu = hc/\lambda$, where h is Planck's constant, and ν and λ are the frequency and wavelength of the radiation.”

Einstein called his packets, “light-quanta”; the modern term, “photon” was coined by G.N.Lewis [2]. This localized-packet concept explains the photo-electric effect by a quasi-chemical equation:



*Correspondent: ghunter@yorku.ca, Chemistry Department, York University.

†now with Optech Inc., 100 Wildcat Rd, Toronto, Canada.

‡Physics Department, York University.

the energy of the emitted electron being equal to the energy of the absorbed photon minus the energy required to remove the electron from the atom within the surface of the solid forming the photocell: This equation (1) predicts the experimentally observed characteristics of the photo-electric effect:

1. all the emitted electrons have the same kinetic energy when the light is monochromatic,
2. this kinetic energy increases as the frequency of the light increases (as the wavelength decreases),
3. there is a minimum frequency (ν_0) of the light (maximum wavelength) below which no electrons are emitted; the energy, $h\nu_0$ is the energy required to remove an electron from the surface of the photocell (binding energy of the electron, ionization energy of the atom, within the solid surface),
4. the kinetic energy of the emitted electrons is $h\nu - h\nu_0$, which is observed as the voltage of the photo-cell being given by: $V = (h\nu - h\nu_0)/e$ where e is the electric charge of the electron,¹
5. the electric current generated by the photocell (rate of charge flow \equiv electrons per unit time) is proportional to the radiation intensity absorbed by the cell (equivalent to number of photons per unit area of cell surface absorbed in unit time).

1.1 The Photon in Quantum Field Theory and the Bohr Model

Although the quantization of the radiation field in terms of photons of energy $h\nu$ became part of the standard language of quantum optics [3, 4], Einstein's original concept of the photon as a localized packet of electromagnetic radiation was discarded with the ascendancy of quantum mechanics in the mid-1920s.² Theories of the wave function of the photon [5, 6] preclude this localization, and even the quantum theory of the photo-electric effect models the light as a plane wave [7, pp.215-224].

In contrast, the Bohr model of the photon predicts the size and shape of photons, and is thus a quantification of Einstein's localized packet concept. This prediction of size and shape was not an primary objective of the Bohr model of the photon [8]; the size and shape resulted from imposition of the principle of causality on the chosen solutions of Maxwell's equations. The result was that a circularly polarized photon is a monochromatic electromagnetic traveling wave confined within a circular ellipsoid of length equal to the wavelength (λ), and of diameter λ/π ; i.e. an egg-shaped solitary wave propagating along the long axis of the ellipsoid.³ This prediction of size and shape was most important because it provided a basis for comparison with experimental observations. The comparison produced agreement between the predicted size and shape and those inferred from several experimental measurements and observations. This agreement makes the Bohr model worthy of serious consideration even though its theoretical basis (a quantized solution of Maxwell's equations confined by causality) is dissimilar from the widely accepted quantum field theory of light [3, 4].

This ellipsoidal soliton model of the photon is a Bohr model in the sense that it is a solution of the classical equations of motion that is subsequently quantized. In Bohr's well-known model of the hydrogen atom the classical equations are Newton's equations for the motion of an electron within the field of a proton, whereas for the photon (light regarded as electromagnetic radiation) the appropriate classical equations are Maxwell's equations in vacuum. In Bohr's model of the hydrogen atom the quantization makes the angular momentum of the electron an integer multiple of Planck's constant, $\hbar = h/2\pi$. In the Bohr model of the photon the quantization of the photon's angular momentum

¹ ν_0 is measured as the reverse (stopping) voltage, $(h\nu_0)/e$, needed to just stop the flow of current from the photocell.

²A corollary of Einstein's localized packet concept of the photon is that much of the cross-sectional area of a beam of light is empty space, the proportion of the beam's area occupied by packets increasing with increasing light intensity.

³The ellipsoid is 3 times (accurately π times) as long as its diameter.

arises from an appropriately chosen solution of Maxwell's equations: in addition, the energy of the oscillating electromagnetic field (integrated over the volume of the ellipsoid) is quantized to be $h\nu$ - the known energy of the photon; this quantization fixes the amplitude of the wave, and is analogous to the imposed quantization of angular momentum in Bohr's model of the hydrogen atom; the analogy extends to quantization of the energy being $n h\nu$ with $n > 1$ representing a multiphoton.⁴

The full theoretical derivation of the Bohr model is presented in [8] together with supporting experimental evidence; here the theory and experimental support is summarized and augmented by recent ideas pertaining to how a solitary wave can exhibit two-slit interference.

1.2 The Bohr Model of the Photon Summarized

The solution of Maxwell's equations was chosen to be a monochromatic traveling wave having the observed angular momentum of the photon; i.e. a spin of $\pm\hbar$; constant parameters multiplying each of these spin states allows for representation of all the known polarization states of light.

The chosen solution of Maxwell's equations is confined within a finite space-time region by the principle of Special Relativity that causally related events must be separated by time-like intervals. With the idea that a photon is self-causing as it propagates, causality imposes the condition that events within the wave having the same phase must be separated by time-like intervals. In the limit where the interval becomes null (light-like), causality leads to the inference that the length of the photon along its axis of propagation is the wavelength, λ .⁵ In addition, for circularly polarized states the causally connected field is contained within a circular ellipsoid with maximum diameter (transverse to the axis of propagation) of λ/π ; the length of the ellipsoid (along the axis of propagation) is the wavelength.⁶

This modeling of the photon as an ellipsoidal soliton arises from the imposition of causality upon the solution of Maxwell's equations (which are linear and homogeneous) whereas non-relativistic solitons arise as solutions of non-linear differential equations [9].

The size and shape of the soliton allowed for quantization of its energy; the wave's electromagnetic energy, $\mathbf{E}^2 + \mathbf{H}^2$, integrated over the volume of the ellipsoid, was set to $h\nu$.⁷ This fixed the amplitude of the wave and led to an expression for the average intensity within the photon-soliton [8, eqn.57]:⁸

$$I_p = \frac{4\pi\hbar c^2}{\lambda^4} \quad (2)$$

1.3 Experimental Confirmation of the Soliton

Experiment confirms the predicted size, shape and intrinsic intensity of the photon:

- its length of λ is confirmed by:
 - the generation of laser pulses that are just a few periods long;
 - for the radiation from an atom to be monochromatic (as observed), the emission must take place within one period, τ , [10];

⁴Observations indicate that multiphotons have a strong tendency to separate laterally into single photons moving along parallel propagation axes. This instability and the stability of a photon with just one $h\nu$ of energy is an outstanding mystery of physics, whose eventually resolution should yield a profound insight into the nature of Planck's constant.

⁵or equivalently in time, the period of oscillation $\tau = \nu^{-1}$.

⁶The ellipsoidal soliton can be visualized as an egg, or as an american/rugby football.

⁷Or in general, to $n h\nu$, analogous with Bohr's quantization of angular momentum as $n\hbar$.

⁸The photon's intrinsic intensity is not uniform: being proportional to the radius (r) squared it is zero on the axis of propagation and maximal at the ellipsoid's maximum radius of $r = \lambda/2\pi$.

- the sub-picosecond response time of the photoelectric effect [11];⁹
- the diameter of λ/π is confirmed by:
 - the attenuation of direct (undiffracted) transmission of circularly polarized light through slits narrower than λ/π : our own measurements of the effective diameter of microwaves [8, p.166] confirmed this within the experimental error of 0.5%;
 - the resolving power of a microscope (with monochromatic light) being “a little less than a third of the wavelength”; λ/π is 5% less than $\lambda/3$, [12];
- The predicted intrinsic intensity (given by eqn.2) is the threshold (minimum) intensity to which a laser beam must be focussed in order to produce multiphoton absorption: two experiments confirming this (one with 650nm light [13], the other with $\lambda=10.5\mu m$) are described in [8, p.165].

1.4 Solution of Maxwell’s Equations: the Photon’s Wave Function

Maxwell’s equations [14] relate the first derivatives of the six components of the electromagnetic field; they comprise eight partial differential equations which must be satisfied simultaneously.¹⁰ The key to finding appropriate solutions, is to differentiate to produce second derivatives followed by elimination of common terms between the resulting equations to yield the result that each Cartesian component of the field ($E_x, E_y, E_z, H_x, H_y, H_z$) separately satisfies d’Alembert’s wave equation [14].¹¹

For a wave traveling parallel to the z -axis at the speed of light, c , the solution must be any function of $z - ct$ [15], and if this wave is monochromatic the functional form is simply:¹²

$$S(z - ct) = \exp\{2\pi i(z - ct)/\lambda\} \quad (3)$$

When this form is adopted as a factor of the solution, insertion into d’Alembert’s equation causes a complete separation of z and t from the transverse coordinates ($x = r \cos \phi$, $y = r \sin \phi$),¹³ plane polar coordinates (r, ϕ) being chosen in preference to the Cartesian coordinates (x, y) in view of the axial symmetry of the direction of propagation.

Separation of the radius, r , from the polar angle, ϕ , produces the two ordinary differential equations:

$$\frac{1}{\Phi(\phi)} \frac{d^2 \Phi(\phi)}{d\phi^2} = m^2 = -\frac{1}{R(r)} \left\{ \frac{d^2 R(r)}{dr^2} + \frac{1}{r} \frac{dR(r)}{dr} \right\} \quad (4)$$

where m^2 is the real separation constant introduced to separate r from ϕ .

The simplest solution of eqn.(4) is the plane wave ($m^2 = 0$); i.e. $R(r)$ and $\Phi(\phi)$ both being constants.¹⁴ However, this solution was rejected as unphysical because light is observed to travel along very narrow beams.¹⁵

⁹The predicted absorption time of the ellipsoidal photon is its period of oscillation, $\tau = 1/\nu$ = the transit time of the ellipsoid past any point in space = the time to enter the surface of the photocell: a few femtoseconds for visible light.

¹⁰The equations are linear and homogeneous with constant coefficients.

¹¹This only pertains for the *Cartesian* components; it does not prevail for the spherical or cylindrical components.

¹² $S(z-ct)$ is an eigenfunction of Schrödinger operators: momentum in the direction of propagation, $\hat{p}_z = \frac{\hbar}{i} \frac{\partial}{\partial z}$, with eigenvalue \hbar/λ , and energy, $\hat{E} = -\frac{\hbar}{i} \frac{\partial}{\partial t}$, with eigenvalue $\hbar c/\lambda = h\nu$, the physically known values for the photon.

¹³The separation is complete: there is no separation constant between the z, t and the r, ϕ differential equations.

¹⁴Plane waves are widely used in the quantum field theory of light [3, 4, 16].

¹⁵A plane wave has field components that have the same value throughout any plane perpendicular to the axis of propagation, and thus it is completely non-localized, contrary to observation that light moves along very narrow beams.

The next simplest solution of eqns.(4) is for $m^2 = 1$: i.e. a factor of r or $1/r$, with an angular factor of $\exp\{i(\phi)\}$ or $\exp\{-i(\phi)\}$. These angular factors are eigenfunctions of the z -component of angular momentum, $\mathbf{L}_z = \frac{\hbar}{i} \frac{\partial}{\partial \phi}$, in Schrödinger quantum mechanics [17, p.217], the eigenvalues of $\pm\hbar$ being those observed for the spin angular momentum of the photon; thus these solutions for $m^2=1$ are appropriate for the wavefunction of the photon:

$$\psi(r, \phi, z-ct) = (\alpha r + \beta/r) (A \exp\{i\phi\} + B \exp\{-i\phi\}) \exp\{2\pi i(z-ct)/\lambda\} \quad (5)$$

Having determined this (the form in eqn.5) as the appropriate solution of d'Alembert's equation, each of the 6 field components ($E_x, E_y, E_z, H_x, H_y, H_z$) will have this form, the coefficients (α, β, A, B) being different in each component. The relationships between the coefficients of different components were determined by Maxwell's equations. This produced the inferences:

$$\begin{aligned} E_z &= H_z = 0 && \text{(no field along the axis of propagation : a transverse wave)} \\ E_x &= (\alpha r + \beta/r) (A \exp\{i\phi\} + B \exp\{-i\phi\}) \exp\{2\pi i(z-ct)/\lambda\} = \mu_0 c H_y \\ E_y &= i(\alpha r - \beta/r) (A \exp\{i\phi\} - B \exp\{-i\phi\}) \exp\{2\pi i(z-ct)/\lambda\} = -\mu_0 c H_x \end{aligned} \quad (6)$$

Imposition of the causality condition led to the result that if A or B is zero, then the field must be contained within a circular ellipsoid of length λ and cross-sectional diameter λ/π [8, §2.5].

Since Maxwell's equations are linear and homogeneous they do not determine the amplitude of the solutions; this was determined by integration of the energy of the wave, $\mathbf{E}^2 + \mathbf{H}^2$.¹⁶ This led to the realization that the form $1/r$ would cause a divergent contribution to the energy at $r = 0$, while the form r would cause a similar divergence as $r \rightarrow \infty$. Thus, in view of the causality condition limiting the domain of the field to an ellipsoid along the axis of propagation, it was decided to discard the $1/r$ form and retain the r form in order to produce a finite integrated energy. This discarding of the $1/r$ term (i.e. $\beta=0$ in eqn.6) was concordant with the need to make the field an eigenfunction of L_z [8, §2.6].

This normalization of the amplitude of the photon's field yielded:¹⁷

$$A^2 + B^2 = 1 \quad \text{and} \quad \alpha^2 = 120 n h c \pi^4 / (\epsilon_0 \lambda^6) \quad (7)$$

1.5 The Soliton's Evanescent Wave

An evanescent wave outside the ellipsoid is necessary as an adjunct to the theory presented in [8], because while the relativistic principle of causality confines the wave within the ellipsoid, the radial dependence of the wave within the soliton is simply r , which is a maximum at the surface of the ellipsoid; physically the wave cannot sharply cut-off to zero at this surface; it must smoothly decay towards zero outside the ellipsoid; an evanescent wave decays in this way [7, pp.103-108].

The radial dependence of the evanescent wave is $1/r$; i.e. the solution of Maxwell's equations (eqn.6) with $\alpha=0$. The intensity of this wave decreases as $1/r^2$ as the distance, r , from the axis increases.

J.J. Thomson derived the same solution (eqn.6) of Maxwell's equations in 1924 [18]; he noted that a radial dependence of r is appropriate near $r = 0$, with $1/r$ being appropriate as $r \rightarrow \infty$, but he didn't pursue his analysis as far as deducing an ellipsoidal soliton, with the wave having the r form within the ellipsoid, and the $1/r$ form outside the ellipsoid.

¹⁶This is analogous with Bohr's quantization of the electron's angular momentum in his model of the hydrogen atom.

¹⁷In [8] the amplitude squared ($\alpha^2 = S_0^2$ in [8, eqn.47]) was given as, $\alpha^2 = 64 n h c \pi^4 / (\epsilon_0 \lambda^6)$, which corresponds to integration over a cylinder (length λ and diameter λ/π) rather than the ellipsoid; the factor of 120 in eqn.(7) is correct for integration over the ellipsoid; the relation $A^2 + B^2 = 1$ was imposed, with α^2 determined by the energy integral.

The r dependence within the ellipsoid and the $1/r$ dependence outside the ellipsoid, makes the r -derivative of the wave discontinuous on the surface of the ellipsoid. While this may appear to be unphysical, it is the same discontinuity exhibited by the gravitational force due to the mass of the Earth: on the assumption of a uniform density, the gravitational force inside the Earth is proportional to the radius, r , whereas outside the Earth it decreases like $1/r^2$ [19].

1.6 Characteristics of the Photon's Evanescent Wave

The polar components of the evanescent field are given by eqns.(38) of [8] for $\alpha = 0$ and β given by eqn.(10), which show that none of these components have any dependence upon the polar angle ϕ , and that E_r and H_ϕ are real, while H_r and E_ϕ are imaginary:

$$E_r = \frac{\beta}{r} [A + B] = \mu_0 c H_\phi \quad E_\phi = -i \frac{\beta}{r} [A - B] = -\mu_0 c H_r \quad (8)$$

Independence of the angle, ϕ , means that the evanescent wave carries none of the angular momentum of the photon,¹⁸ and hence none of its energy; it is a truly evanescent wave [7, pp.105-108].

1.7 Matching the Soliton and Evanescent Waves

While the gradient of the wave has a cusp at $r = \lambda/(2\pi)$, the amplitude must be continuous at $r = \lambda/(2\pi)$; equating of the soliton and evanescent wave amplitudes at $r = \lambda/(2\pi)$ produces:

$$\alpha r = \beta/r \quad \text{for} \quad r = \lambda/(2\pi) \quad (9)$$

and since α^2 is given by eqn.(7) it follows that:

$$\beta^2 = [\lambda/(2\pi)]^4 \times 120nhc\pi^4/(\epsilon_0\lambda^6) = 7.5nhc/(\epsilon_0\lambda^2) \quad (10)$$

Orthogonality of the Radial Gradients The radial gradient of the soliton wave is simply the normalization constant, α , while that of the evanescent wave is $-\beta/r^2$. Thus at the cusp where the two waves join (at $r = \lambda/(2\pi)$) the ratio of these gradients is:

$$\text{ratio of gradients} = -\frac{\beta}{\alpha r^2} = -1 \quad \text{at} \quad r = \lambda/(2\pi) \quad (11)$$

Thus where the soliton and evanescent waves meet (at $r = \lambda/(2\pi)$) they are orthogonal to each other - independent of the wavelength, λ .

The above matching of the soliton and evanescent waves was made at the soliton's maximum diameter of λ/π ; this raises the question of their matching at values of z other than $z=0$; i.e. at other points on the ellipse:

$$\begin{aligned} (2\pi r)^2 + (2z)^2 &= \lambda^2 \\ \text{i.e. when } r &= \frac{1}{2\pi} \sqrt{(\lambda)^2 - (2z)^2} \quad \text{for} \quad -\frac{\lambda}{2} < z < +\frac{\lambda}{2} \end{aligned} \quad (12)$$

It might appear natural to apply the matching condition of eqn.(9) for all values of r specified in eqn.(12) to produce:

$$\begin{aligned} \beta^2 &= \left[\frac{1}{2\pi} \sqrt{(\lambda)^2 - (2z)^2} \right]^4 \times 120nhc\pi^4/(\epsilon_0\lambda^6) \\ &= \left[(\lambda)^2 - (2z)^2 \right]^2 \times 7.5nhc/(\epsilon_0\lambda^6) \end{aligned} \quad (13)$$

¹⁸Because the operator for the z -component of angular momentum is $\mathbf{L}_z = \frac{\hbar}{i} \frac{\partial}{\partial \phi}$.

This would have the effect of making the amplitude of the evanescent wave, β , smaller as z changes from $z=0$ to $z = \pm \frac{\lambda}{2}$, with β being zero at these limits (the ends of the ellipsoid).

However, this conjecture would make β a function of z (as in eqn.13) rather than a constant, and hence the evanescent field (eqn.6 for $\alpha=0, \beta \neq 0$) would no longer be a solution of Maxwell's equations. The resolution of this physical vs. mathematical paradox may be found within the framework of General Relativity, in which the photon's local energy produces a non-Lorentzian metric.

1.8 Diffraction and Interference

The evanescent wave is believed to be responsible for the phenomena of diffraction and interference. As a photon-soliton passes close to the edge of, or through a slit in, a material obstacle placed within the beam of light, the interaction between the electrons within the obstacle and the photon's evanescent wave will cause its path to bend as it passes by, the angle of bending (diffraction) being dependent upon the impact parameter of the soliton's axis with the edge or slit.

Double slit interference can be understood by the soliton itself (like the C_{60} molecules in Zeilinger's experiment [20]) going through one slit or the other, while its evanescent wave extends over both slits. The evanescent wave is like a classical continuous wave in extending throughout all space, and hence the interference minima and maxima will appear at the same positions as predicted by Huygen's theory. However, the soliton model predicts that:

- the individual photons will arrive at local positions in the detection plane, whereas the classical continuous wave model predicts a uniformly visible interference pattern: that the former (rather than the latter) is actually observed supports the soliton model [20];
- the visibility of the interference pattern¹⁹ will decrease with slit separation (because the intensity of the evanescent wave decreases like $1/r^2$, r being the distance from the soliton's axis of propagation), whereas the classical continuous wave model predicts a visibility independent of slit separation. This seems not to have been investigated experimentally [3, 7, 21].

A double-slit experiment by Alkon [22] exhibits the expected interference pattern even though the individual photons are constrained to pass through one slit or the other by an opaque barrier extending from the source (a laser) up to the mid-point between the slits.²⁰ This experiment demonstrates that the particle-like photon (the Bohr model soliton) passes through one slit or the other, and yet its passage through this slit (and the subsequent diffraction) is affected by the presence of the other slit; this effect of the other open slit is evidence for the existence of the evanescent wave surrounding the soliton.²¹

A causal model of diffraction has been proposed by Gryzinski [24]; it is based upon the photon being a particle-like (localized) electromagnetic wave that interacts with the array of positive atomic nuclei and negative electrons within a solid, as it passes:

- through a crystal (Bragg diffraction of X-rays), or
- adjacent to an edge of a sheet of the solid (an edge of a slit).

¹⁹Visibility, V , is defined by: $V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$, I_{\max} and I_{\min} being the measured intensities at the interference maxima and minima respectively; it has the range: $0 \leq V \leq 1$.

²⁰Alkon's experiment is the experimental proof that the continuous wave concept that "the photon goes through both slits and interferes with itself" is not correct.

²¹Interaction between the evanescent waves of collaterally moving photon-solitons could be the cause of the very small (but finite) divergence of a laser beam [23, p.6].

Gryzinsky's model of diffraction does not specify the size or shape of the soliton, but it quantitatively explains both Bragg diffraction and double-slit interference; his concept of the latter is that while the localized photon goes through one slit, its wave extends to the other slit. His theory is concordant with the Bohr model's evanescent wave, specifically because his localized model involves the concept that "the photon's electric field decreases when distance [from its center] increases".

Gryzinsky pertinently cites Zeilinger's observation that each photon manifests its particle (localized) nature in each detection event: the distribution of detection events²² only becomes manifest after a large number ($\geq 10^4$) of detection events have been recorded [20]; each photon detection is a localized event.

The evanescent wave explanation for diffraction and interference is not readily invoked for the Mach-Zender type of interferometer, because the two alternative paths for the photon are typically separated by distances over which the evanescent wave's intensity would have become negligible; a small difference (of the order of the wavelength) between the lengths of the two paths determines the observed interference pattern. This observation requires further theoretical explanation.

References

- [1] A. Einstein, *Ann.der Phys.*, **17**, 132 (1905).
- [2] G.N.Lewis, *Nature.*, **118**, 874 (1926).
- [3] L.Mandel and E.Wolf, *Optical Coherence and Quantum Optics*, Cambridge University Press, 1995.
- [4] M.O.Scully and M.S.Zubairy, *Quantum Optics*, Cambridge University Press, Cambridge, UK, 1997.
- [5] J.Sipe, *Photon Wave Functions*, *Phys.Rev.A*, **52**, 1875 (1995).
- [6] I.Bialynicki-Birula, *Photon Wave Function*, *Progress in Optics*, Vol.XXXVI, Elsevier, Amsterdam, 1996.
- [7] E. Wolf, *Selected Works of Emil Wolf*, World Scientific, Singapore, 2001.
- [8] G.Hunter and R.L.P.Wadlinger, *Photons and Neutrinos as Electromagnetic Solitons*, *Physics Essays*, Vol.2, 158-172 (1989).
- [9] F.Gesztesy, *Soliton equations and their algebro-geometric solutions*, Cambridge University Press, 2003.
- [10] M.Kowalski, *Photon Emission from Atomic Hydrogen*, *Physics Essays*, Vol.12, 312-331 (1999).
- [11] M.C.Downer, W.M.Wood, and J.I.Trisnadi, *Comment on Energy Conservation in the Picosecond and Subpicosecond Photoelectric Effect*, *Phys.Rev.Lett.* **65**, 2832 (1990).
- [12] S.G.Starling and A.J.Woodall, *Physics*, second edition, Longmans Green and Co., London, UK, 1957, p.706.
- [13] R.D.Verma and A.Chandra, *J.Optical Society of America*, **B5**, 86 (1988).

²²attributed in the continuous wave model to the wave going through both slits and self-interfering

- [14] J.H. Fewkes and J. Yarwood, *Electricity and Magnetism*, Univeristy Tutorial Press, London 1956, p.509.
- [15] C.A. Coulson, *Waves*, 7th edition. Oliver and Boyd, London, 1955, pp.2,103,111.
- [16] W. Heitler, *Quantum Theory of Radiation*, Oxford University Press, Oxford, UK, 1954.
- [17] D.A. MacQuarrie, *Quantum Chemistry*, University Science Books, Saualito, CA 94965 (1983).
- [18] J.J. Thomson, *Philos.Mag.Ser.6*, **48**, 737 (1924), & *Nature*. **137**, 23 (1936).
- [19] <http://abel.math.harvard.edu/~knill/math21a/hell.pdf>
- [20] O. Nairz, M. Arndt, and A. Zeilinger, *Quantum Interference Experiments with Large Molecules*, *Am.J.Physics*, **71**, 319-325, (2003).
- [21] M. Born and E. Wolf, *Principles of Optics: Electromagnetic Theory of Propagation, Interference and Diffraction of Light*, Cambridge University Press, 1999.
- [22] D.L. Alkon, “*Either-Or*” *Two-Slit Interference: Stable Coherent Propagation of Individual Photons Through Separate Slits*, *Biophysical Journal*, **80**, 2056-2061 (2001).
- [23] T.P. Softley, *Atomic Spectra*, Oxford University Press, Oxford, UK, 1994.
- [24] M. Gryzinski, *Spin-Dynamical theory of the Wave-Corpuscular Duality*, *International Journal of Theoretical Physics*, **26**, 967-979 (1987).